Teacher Guide: Quadratics in Polynomial Form

Learning Objectives

Students will…

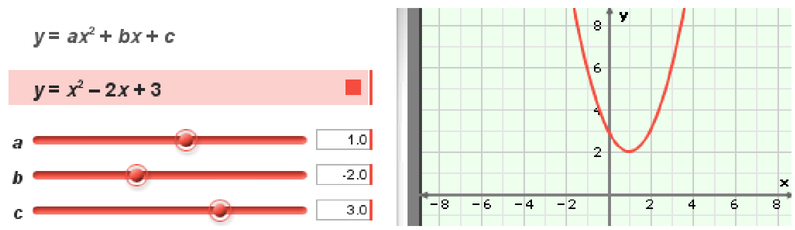
- Discover how the graph of $y = ax^2 + bx + c$ is affected when a, b, and c change.
- Discover how to find the vertex and the equation of the axis of symmetry of the graph of $y = ax^2 + bx + c$.
- Use quadratic functions to solve real-world problems.

Vocabulary

axis of symmetry, parabola, quadratic function, vertex of a parabola

Lesson Overview

In the *Quadratics in Polynomial Form* Gizmo™, students can vary the coefficients in the quadratic equation $y = ax^2 + bx + c$ and see how each value affects the graph.



The Student Exploration sheet contains three activities:

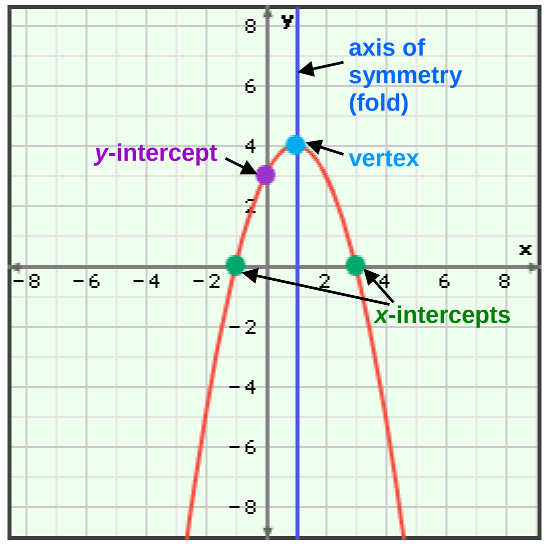
- Activity A – Students discover how the values of a, b, and c affect the graph of the quadratic function $y = ax^2 + bx + c$.
- Activity B – Students explore the intercepts, vertex, and axis of symmetry of quadratic functions, and discover how they are related.
- Activity C – Students use quadratic functions to solve real-world problems.

Suggested Lesson Sequence

1. **Pre-Gizmo activity** (5 – 10 minutes)
   Have students work in small groups or pairs to answer the following questions:
   - How can the graph of $y = 2x – 10$ be used to solve $0 = 2x – 10$? [The x-intercept of the graph is the solution to $0 = 2x – 10$.] Verify your answer by drawing the graph and solving the equation. [The x-intercept and solution are both 5.]
   - Now think about the quadratic equation $0 = x^2 + 4x – 12$. How do you think the graph of $y = x^2 + 4x – 12$ can be used to solve $0 = x^2 + 4x – 12$? [The x-intercepts of the graph are the solutions to $0 = x^2 + 4x – 12$.] You will explore this concept in more detail in Activity B on the Student Exploration sheet.

2. **Prior to using the Gizmo** (10 – 15 minutes)
   Before students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. Afterwards, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations. Show students how to take snapshots in the Gizmo and paste the images into a blank document.

3. **Gizmo activity** (15 – 20 minutes per activity)
Assign students to computers, individually or in pairs. Have students work through the Student Exploration (SE) sheet, using the Gizmo. (Or, you can use a projector and do the SE as a teacher-led activity.) Either way, we recommend doing page 1 of the SE (Prior Knowledge Questions and Gizmo Warm-up) plus one of the SE activities.



ELL Adaptation – Place students with a partner. If possible, pair each ELL student with an English-speaking student. Have each pair graph the quadratic function $y = -x^2 + 2x + 3$ on graph paper. Ask them to fold the paper in half so the two sides of the parabola match. Have them find the equation of the axis of symmetry, draw it on the graph, and write about what they discovered. Then ask them to label the vertex, *x*-intercepts, and *y*-intercept. Encourage students to share their graphs and explanations with the class.

4. **Discussion questions** (15 – 30 minutes)
As students are working or just after they are done, discuss the following questions:

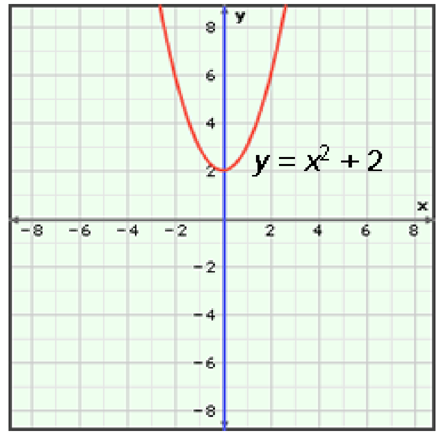
- Suppose the equation of the axis of symmetry of a parabola is $x = -1$ and one of the points on the graph is (–4, –7). What other point must also be on the graph? [(2, –7)] Explain your answer. [Both points have the same *y*-coordinate and are the same distance from the axis of symmetry.]
- Suppose the vertex of a parabola is (–3, –4) and one of the *x*-intercepts is –5. Can you find the coordinates of the other *x*-intercept? [Yes.] Explain. [The axis of symmetry must be $x = -3$. Because –5 is 2 units to the left of the axis of symmetry, the other *x*-intercept must be 2 units to the right of it: $-3 + 2 = -1$.]
- A quadratic function generally has the form $y = ax^2 + bx + c$, where $a \neq 0$. Why is it important to state that the value of *a* cannot be zero? [If $a = 0$, the function would just be $y = bx + c$, which would be a linear function, not quadratic.]

5. **Follow-up activity – Challenge** (15 – 20 minutes)
Have students work in pairs or small groups to solve the following problems:

- In the Gizmo, start with the following quadratic functions, in which $b = 0$. Turn on **Show vertex trail** and use the *b* slider to vary the value of *b*. What's the equation of the vertex trail? (Hint: Compare the vertex trail to the graph you started with.)
  - $y = x^2 - 1$ [Answer: $y = -x^2 - 1$]
  - $y = 2x^2 + 5$ [Answer: $y = -2x^2 + 5$]
  - $y = -x^2 + 7$ [Answer: $y = x^2 + 7$]
  - $y = -4x^2 + 1$ [Answer: $y = 4x^2 + 1$]
- What's the general rule here? In other words, what is the equation of the vertex trail when you graph $y = ax^2 + bx + c$ and then vary the value of *b*? [The equation of the vertex trail of $y = ax^2 + bx + c$ is $y = -ax^2 + c$.]

Mathematical Background

The *polynomial form* (or *standard form*) of a quadratic function is $y = ax^2 + bx + c$, where $a \neq 0$. (Note that if $a = 0$, the equation would not have an x^2 term and therefore would be linear.)



The graph of a quadratic function is a symmetrical curve called a *parabola*. When the leading coefficient is positive ($a > 0$), the graph opens upward and the vertex is a minimum of the function. When $a < 0$, the graph opens downward and the vertex is a maximum of the function.

The values of a, b, and c affect a parabola in different ways. (The Gizmo illustrates this very well.) Varying the value of a vertically *dilates* (stretches) the y-values toward or away from the x-axis. This makes the parabola appear thinner or wider. Varying the value of c *translates* (moves) all the y-values (and hence the whole parabola) up or down, with the same shape.

All parabolas are symmetric about a vertical line called the *axis of symmetry*. If you fold a parabola in half, the two sides will always match. You can think of the axis of symmetry as being the "middle" of the parabola – it's the midpoint of the line segment between each pair of points with the same y-coordinates. (It also lies right on the "fold" line.)

Quadratic functions can also be expressed in *vertex form*, $y = a(x - h)^2 + k$. In this form, (h, k) is the vertex and $x = h$ is the axis of symmetry. A quadratic function given in standard form can be written in vertex form by completing the square as shown in the following example:

$$\begin{aligned} y &= -x^2 - 4x + 8 \\ &= -1(x^2 + 4x) + 8 \\ &= -1(x^2 + 4x + 4) + 8 - (-1)(4) \\ &= -1(x + 2)^2 + 12 \end{aligned}$$

The vertex of $y = -x^2 - 4x + 8$ is $(-2, 12)$ and the equation of the axis of symmetry is $x = -2$.

There are many real-world applications of quadratic functions. For example, when an object is hit or thrown into the air, its height at time t is given by the equation $h = -\frac{1}{2}gt^2 + v_0t + h_0$, where g is Earth's gravitational constant (32 ft/sec^2 or 9.8 m/sec^2), v_0 is the initial upward velocity, and h_0 is the initial height.

So, for a golf ball hit off of the ground with an initial upward velocity of 64 feet per second, $g = 32$, $v_0 = 64$, and $h_0 = 0$. The quadratic equation that describes the height of the golf ball (in feet) in this situation is $h = -16t^2 + 64t$.

Selected Web Resources

Quadratic functions video (Khan Academy): http://www.youtube.com/watch?v=RjkTEyO5Zso
Applications video (Khan Academy): http://www.youtube.com/watch?v=Zoa485PqK_M
Quadratics in Factored Form Gizmo: http://www.explorelearning.com/gizmo/id?115
Quadratics in Vertex Form Gizmo: http://www.explorelearning.com/gizmo/id?150